5/9/18 "I make the most of all that comes and the least of all that goes."-Sara Teasdale

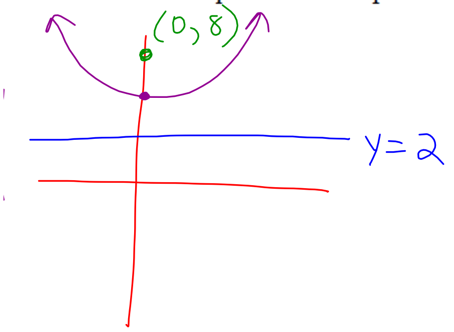
HW: "Average Rate of Change" homework section

Test 2 on Wednesday 5/30

AIM: How do we find Average Rates of Change?

Warm Up:

Determine the equation of the parabola whose focus is (0,8) and whose directrix is the horizontal line y = 2?



Need

Vertex: $\frac{8+2}{2} = 5 \rightarrow (0,5)$

P-value: $\frac{8-2}{2} = 3$ $p = 3$

$$y = +\frac{1}{4(3)}(x-0)^2 + 5$$

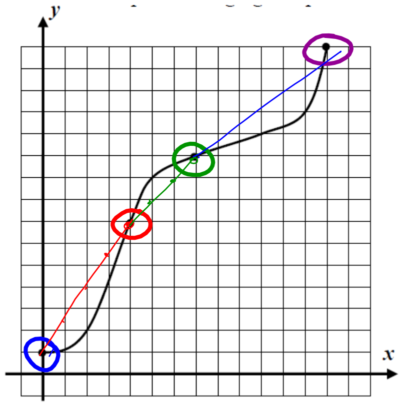
OR

$$y = \frac{1}{12}x^2 + 5$$

***Exercise* #1:** The function $f(x)$ is shown graphed to the right.

(a) Evaluate each of the following based on the graph:

(i) $f(0)$ $\boxed{1}$ (ii) $f(4)$ $\boxed{7}$ (iii) $f(7)$ $\boxed{10}$ (iv) $f(13)$ $\boxed{15}$



(b) Find the change in the function, Δf over each of the following domain intervals. Find this both by subtraction and show this on the graph. y-values "x"

(i) $0 \le x \le 4$

$y=1$ $y=7$

$7-1=6$

$\boxed{6}$

(ii) $4 \le x \le 7$

$y=7$ $y=10$

$10-7=3$

$\boxed{3}$

(iii) $7 \le x \le 13$

$y=10$ $y=15$

$15-10=5$

$\boxed{5}$

(c) Why can't you simply compare the changes in f from part (b) to determine over which interval the function changing the fastest?

The intervals (x-values) are not the Same.

(d) Calculate the **average rate of change** for the function over each of the intervals and determine which interval has the greatest rate. AROC slope $\frac{\Delta Y}{\Delta x}$

(i) $0 \le x \le 4$

$\frac{\Delta f}{\Delta x} = \frac{\Delta Y}{\Delta x}$

$\frac{7-1}{4-0} = \frac{6}{4} = \boxed{1.5}$

Greatest

(ii) $4 \le x \le 7$

$\frac{\Delta f}{\Delta x}$ $\frac{10-7}{7-4} = \frac{3}{3}$

$\boxed{1}$

(iii) $7 \le x \le 13$

$\frac{\Delta f}{\Delta x} = \frac{15-10}{13-7} = \frac{5}{6} = .833$

$\boxed{.833}$

(e) Using a straightedge, draw in the lines whose slopes you found in part (d) by connecting the points shown on the graph. The average rate of change gives a measurement of what property of the line?

The Slope.

AVERAGE RATE OF CHANGE → Average Slope

For a function over the domain interval $a \le x \le b$, the function's **average rate of change** is calculated by:

$f(x) = y$

$$\frac{\Delta f}{\Delta x} = \frac{\text{change in the output}}{\text{change in the input}} = \frac{f(b)-f(a)}{b-a} \qquad \frac{y_2-y_1}{x_2-x_1}$$

Exercise #2: Consider the two functions $f(x) = 5x+7$ and $g(x) = 2x^2+1$.

(a) Calculate the average rate of change for both functions over the following intervals. Do your work carefully and show the calculations that lead to your answers.

(i) $-2 \le x \le 3$

$y = 5x+7$
$= 5(-2)+7 = -3$
$(-2,-3)$

line

$\boxed{AROC = \frac{22-(-3)}{3-(-2)} = 5 \quad y=5x+7}$

$y = 5x+7$
$= 5(3)+7$
$= 22$
$(3,22)$

$g(-2) = 9$
$g(3) = 19$
$(-2,9)\ (3,19)$

$AROC = \frac{19-9}{3-(-2)} = \frac{10}{5} = \boxed{2}$

(ii) $1 \le x \le 5$

$y = 5x+7$ AROC (slope) $= 5$

$g(1) = 2(1)^2+1 = 3 \quad (1,3)$

$g(5) = 2(5)^2+1 = 51 \quad (5,51)$

$AROC = \frac{3-51}{1-5} = \frac{-48}{-4} = \boxed{12}$

(b) The average rate of change for f was the same for both (i) and (ii) but was not the same for g. Why is that?

$f(x) = 5x+7$ (line)
$g(x) = 2x^2+1$ (parabola)

f is a line whose slope is always the same.

Exercise #3: The table below represents a linear function. Fill in the missing entries.

rate of change is same

Slope $= \frac{y_2-y_1}{x_2-x_1}$

x	1	5	11	b	45
y	-5	1	a	22	c

Slope $= \frac{1-(-5)}{5-1} = \frac{6}{4} = \frac{3}{2}$

Find b

$\frac{3}{2} = \frac{1-22}{5-b}$

$\frac{3}{2} = \frac{-21}{5-b}$

$15-3b = -42$
$-15 \qquad -15$

$\frac{-3b}{-3} = \frac{-57}{-3}$

$\boxed{b = 19}$

$\frac{3}{2} = \frac{-5-a}{1-11}$

$\frac{3}{2} = \frac{-5-a}{-10}$

$-30 = -10-2a$
$+10 \qquad +10$

$\frac{-20}{-2} = \frac{-2a}{-2}$

$\boxed{10 = a}$

To find:

$\frac{3}{2} = \frac{1-c}{5-45}$

$\frac{3}{2} = \frac{1-c}{-40}$

$\frac{-120}{-2} = \frac{2-2c}{-2}$

$\frac{-122}{-2} = \frac{-2c}{-2}$

$\boxed{61 = c}$